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The third family of compact stars with the color-flavor locked quark core

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In this paper, we study the third family of compact stars with the color-flavor locked (CFL) quark core. The relativistic mean field model is used for hadronic matter and the MIT bag model for CFL quark matter. The results of the calculation show a transitional behavior that goes from the hadron star range, through the transition range, into the CFL quark star range. In the transition range, the third family of compact stars with the CFL quark matter core is found in the wide range of the CFL energy gap 100 MeV⩽Δ<150 MeV. By comparing with early investigations, we argue that the greatest possible third family of compact stars may be the hybrid stars with the CFL quark core.

the third family of compact stars, color-flavor locked quark core, the mass-radius relation, hybrid star

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Neutron stars are natural laboratories to study the properties of compact matter [1,2]. In the interior of a neutron star there exist phase transitions from hadronic matter to various exotic matter, such as hyperonic matter, meson condensation and quark matter [3–5]. It is widely accepted that hadronic matter undergoes a phase transition to strange quark matter at the high density range [6,7]. The study of quantum chromodynamics (QCD) indicates that quark matter might be in a color superconducting phase at quite high density range [8,9]. The essence of color superconductivity is based on the Bardeen, Cooper, and Schrieffer (BCS) pairing mechanism [10]. Theoretical researchers generally agree that the ground state of QCD with three flavors is the color-flavor-locked (CFL) phase [11,12]. At present, the hybrid stars with a CFL quark matter core have been extensively studied [13–15].

Two families of compact stars, white dwarfs and neutron stars, are known well. Almost 40 years ago, Gerlach [16] found that a third family of stable configurations of compact stars could exist in nature besides two families of white dwarfs and neutron stars. He noted that it was due to a large discontinuous behavior in the speed of sound ($dp/d\varepsilon$) of the corresponding equation of state (EoS). Glendenning [17] also found that the practical physical mechanism was a phase transition from hadronic to quark matter. Schertler et al. [18] investigated the phase transition from hadronic to normal quark matter and the possibility of the third family of compact stars. They concluded that the third family could serve as a signature for a phase transition from hadronic to quark matter.

This paper will investigate the influence of the CFL energy gap on the bulk properties of neutron stars and discuss

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the range of the CFL energy gap that exist in the third family of compact stars (in our calculation), and present the possibility of the appearance of the non-identical neutron star twins with the same EOS.

The relativistic mean field theory (RMFT) [19] with the NLSH parameter set is adopted to describe the hadronic matter phase (HP) which only consists of protons, neutrons, and electrons. The free energy is used to describe the CFL quark matter phase, $\Omega_{\rm CFL}$, in the MIT bag model [20]:

$$\Omega_{\rm CFL}=\Omega_{\rm CFL}^{\rm quarks}+\Omega_{\rm CFL}^{\rm Gb}+\Omega^{\rm electrons}, \quad (1)$$

where the first term in eq. (1) is from the quarks, the second term is from the Goldstone bosons, and the third term is from the electrons, respectively. The $\Omega_{\rm CFL}^{\rm quarks}$ term is obtained as [21]

$$\Omega_{\rm CFL}^{\rm quarks}=\Omega_{\rm free}-\frac{3}{\pi^2}\Delta^2\mu^2+B, \quad (2)$$

where the first term in eq. (2) is the free energy of the non-interacting (unpaired) quarks. All quarks have a common Femi momentum ν and intend to pair; hence they have the same quark number density n ($=n_{\rm u}=n_{\rm d}=n_{\rm s}$). $\Omega_{\rm free}$ is given by

$$\Omega_{\rm free}=\frac{6}{\pi^2}\int_0^{\nu}(p-\mu)p^2{\rm d}p+\frac{3}{\pi^2}\int_0^{\nu}(\sqrt{p^2+m_{\rm s}^2}-\mu)p^2{\rm d}p, \quad (3)$$

where μ is the quark number chemical potential, $\mu=(\mu_{\rm u}+\mu_{\rm d}+\mu_{\rm s})/3$. $m_{\rm s}$ is the strange quark mass (here u, d quark masses are neglected). The common Fermi momentum and the same quark number density can be written as

$$\nu=2\mu-\sqrt{\mu^2+\frac{m_{\rm s}^2}{3}}, \quad (4)$$

$$n=\frac{\nu^3+2\Delta^2\mu}{\pi^2}, \quad (5)$$

where Δ is the CFL energy gap. The second term in eq. (2) is from CFL pairs, while the third term is the bag constant.

The appearance of the second term, $\Omega_{\rm CFL}^{\rm Gb}$ in eq. (1) is due to the fact that the chiral symmetry is broken in the CFL phase. The threshold condition of negative kaon, k^- (negative pion, π^-) condensation is $m_{k^-}=\mu_{\rm e}$ ($m_{\pi^-}=\mu_{\rm e}$), where m_{k^-} (m_{π^-}) is the effective mass of k^- (π^-) mesons. However, kaons are more favorable because the effective mass of kaons is lighter than that of pions. Thus, the free energy associated with k^- condensation is obtained by

$$\Omega_{\rm CFL}^{\rm Gb}=-\frac{1}{2}f_\pi^2\mu_{\rm e}^2\left(1-\frac{m_k^2}{\mu_{\rm e}^2}\right)^2, \quad (6)$$

where the decay constant and the effective mass of kaons are given in [13].

Finally, the third term in eq. (1) stands for the contribution of electrons, expressed by

$$\Omega^{\rm electrons}=-\frac{\mu_{\rm e}^4}{12\pi^2}. \quad (7)$$

In the MIT bag model, the energy density and pressure of CFL quark matter can be written as

$$\varepsilon_{\rm CFL}=\sum_{i={\rm u,d,s,e}}\mu_i n_i+\Omega_{\rm CFL}, \quad (8)$$

$$P_{\rm CFL}=-\Omega_{\rm CFL}, \quad (9)$$

where $n_{\rm e}=-\dfrac{\partial\Omega_{\rm CFL}}{\partial\mu_{\rm e}}$.

The preceding discussion demonstrated that the CFL quark phase is negatively charged due to the negative kaon condensation. According to Glendenning construction, it should exist the mixed phase (MP), which contains positively charged hadronic matter coexisting with negatively charged CFL quark matter [14]. μ and $\mu_{\rm e}$ are chosen as independent components. In term of the Gibbs condition of phase equilibrium, we get

$$P_{\rm HP}(\mu,\mu_{\rm e})=P_{\rm CFL}(\mu,\mu_{\rm e})=P_{\rm MP}. \quad (10)$$

In the mixed phase, the neutron and quark chemical potential satisfy

$$\mu_{\rm n}=3\mu. \quad (11)$$

The mixed phase satisfies global charge neutral condition [3]. Thus we have

$$q_{\rm MP}=(1-\chi)q_{\rm HP}+\chi q_{\rm CFL}=0, \quad (12)$$

where χ is the volume fraction of the CFL quark matter, and global baryon number conservation is imposed through [14]

$$n_{\rm B}^{\rm MP}=(1-\chi)n_{\rm B}^{\rm HP}+\chi n_{\rm B}^{\rm CFL}. \quad (13)$$

The total energy density in the mixed phase is

$$\varepsilon_{\rm MP}=(1-\chi)\varepsilon_{\rm HP}+\chi\varepsilon_{\rm CFL}. \quad (14)$$

Solving self-consistently nonlinear transcendental equation sets mentioned above (eqs. (10)–(14)), we can get the total EOS of hybrid stars.

In our calculations, the NLSH parameter set is used in the framework of the RMFT to describe hadronic matter, and the MIT bag model is used to describe CFL quark matter. We choose the strange quark mass $m_{\rm s}$=150 MeV, the bag constant $B^{1/4}$=190 MeV, to study the interaction of quarks, the CFL energy gap Δ changes from 0 to 200 MeV.

Figure 1 presents the EOS of hybrid stars with the CFL quark matter cores. The hadronic part of the EOS is the NLSH EOS, the CFL energy gap Δ changes from 100 to

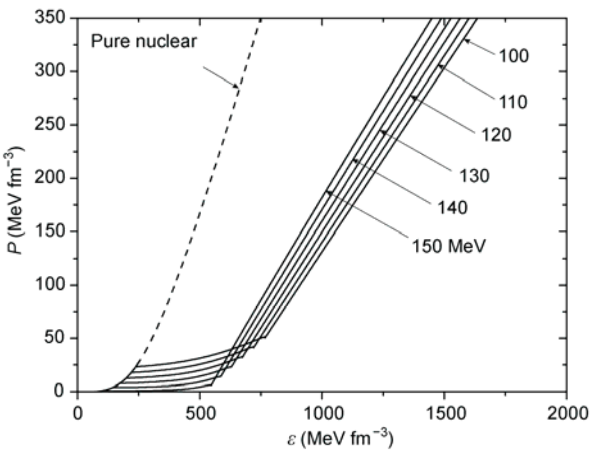

Figure 1 The EOS with the CFL energy gap Δ=100–150 MeV. The dashed line denotes the EOS of pure nuclear matter.

150 MeV. From Figure 1, it can be found that the speeds of sound ($dp/d\varepsilon$) of the EOS are discontinuous at the start points of the phase transition from hadronic to the CFL quark matter, which indicates the third family of compact stars could appear. With the hybrid star EOS shown in Figure 1, the corresponding mass-radius relations of the hybrid stars can be obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [3].

Figure 2 demonstrates the mass-radius relations with the change of the CFL energy gap, Δ=0–200 MeV. It also shows an interesting phenomenon. When the CFL energy gap increases from 0 to 100 MeV, the maximum mass of hybrid star decreases from 2.01$M_\odot$ to 1.32$M_\odot$ ($M_\odot$ is the solar mass), the corresponding radius is from 14.43 to 13.89 km, and the behavior of the hybrid stars are close to hadron stars. When the energy gap Δ>150 MeV, the maximum mass of star in this range increases from 1.44$M_\odot$ to 1.67$M_\odot$, the corresponding radius is from 7.56 to 8.45 km, and the stars in this range show the behavior of CFL quark stars.

While in the range of 100 MeV⩽Δ<150 MeV, the hybrid stars show a transitional behavior from hadron stars to CFL quark stars. And in this range, it can be found that after neutron star branch, there is an unstable region followed by another stable compact star branch; it is so-called the third family (TF) branch of compact stars [18]. In our model, it can be found that the CFL energy gap range of the existing third family of compact stars is between 100 and 150 MeV. In Figure 2, we also plot the end point of the mixed hadronic and CFL quark matter phase (unfilled circles). It implies that the hybrid stars in the third family branch contain a pure CFL quark matter core surrounded by a layer of the mixed hadronic and CFL quark matter phase. However, the hybrid stars in neutron star branch only possess the mixed phase core. We want to emphasize that our calculation results of the third family of hybrid stars with the CFL quark core appear in the wide range of the CFL energy gap (100 MeV⩽Δ<150 MeV). Using the TM2 EOS, the early studies [18,22] proved that the third family of hybrid stars with the normal quark matter core exists in the narrow parameter range of 176 MeV⩽$B^{1/4}$<182 MeV. With NLSH EOS, we obtain a similar result by calculating the mass-radius relations of the hybrid stars with a normal quark matter core. The result is shown in Figure 3 which demonstrates that the third family of compact stars only exists in the very narrow parameter range of 173 MeV⩽$B^{1/4}$<179 MeV.

Because of partial overlapping mass regions of the neutron star branch and the third family branch, non-identical stars having the same mass with different composition and radii can exist. Such a pair of compact stars is called neutron star twins. We can observe the two stars with almost the same mass but different radii as the possibility of the existence of a third family of compact stars. Therefore, we can identify a third family of compact stars by measuring the mass and the radius of neutron stars. Figure 4 presents

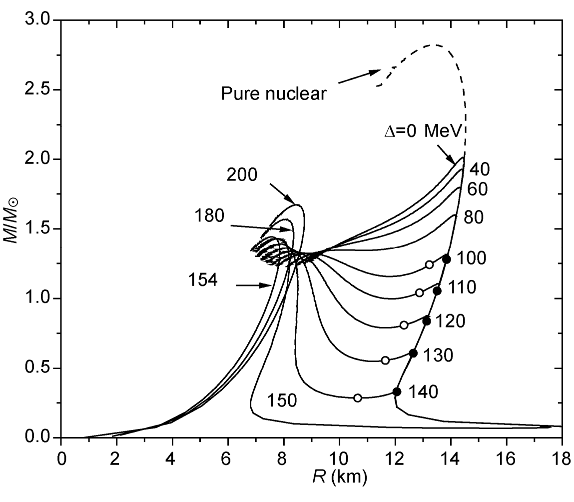

Figure 2 The mass-radius relations for the NLSH parameter set with the energy gap Δ=0–200 MeV, m_s=150 MeV, $B^{1/4}$=190 MeV. The dashed line denotes pure nuclear matter. The filled (unfilled) circles on some curves denote the onset (end) points of the mixed phase.

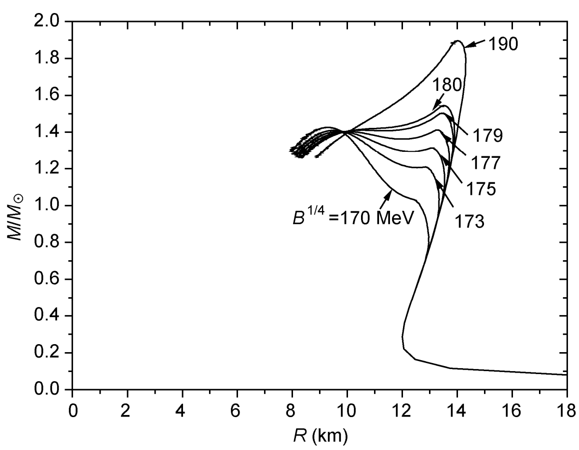

Figure 3 The mass-radius relations for phase transition from hadronic matter to normal quark matter for the NLSH parameter set with m_s=150 MeV and $B^{1/4}$=170–190 MeV.

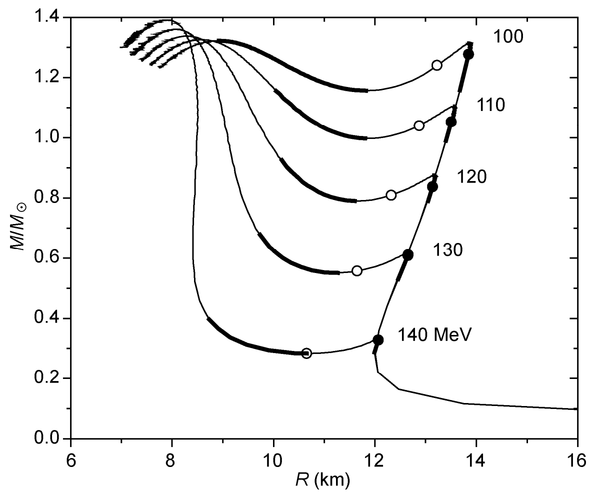

Figure 4 The mass-radius relations for hybrid stars with the CFL energy gap Δ=100–140 MeV. Two thick solid lines on each curve represent the range of high density twin and low density twin, respectively. The filled (unfilled) circles are same as Figure 2.

the mass-radius relation for hybrid stars with the CFL energy gap Δ=100–140 MeV, where two thick solid lines represent the range of high density twin and low density twin, respectively. From Figure 4 it can be found that neutron star twins always exist if a third family appears; in turn, neutron star twins are treated as a signature for the existence of a third family.

In this paper, the MIT bag model for the quark phase and the RMFT for the hadronic phase are used to investigate the third family of compact stars with the CFL quark matter core. In our calculation, with the change of the CFL energy gap we find the behavior of hybrid stars that goes from the hadron star range (Δ=0–100 MeV), through the transition range (100 MeV⩽Δ<150 MeV), into the CFL quark star range (150 MeV⩽Δ<200 MeV). The calculation also shows that neutron star twins always exist in certain region, and they are treated as a signature for the existence of a third family. In addition, in the transition range we find the third family of hybrid stars with the CFL quark core in the wide range of the CFL energy gap (100 MeV⩽Δ<150 MeV). In the early investigation, among 200 mass-radius relations the only third family of hybrid stars with the normal quark core was found using the TM2 EOS in the narrow parameter range (176 MeV⩽$B^{1/4}$<182 MeV). Using the NLSH EOS we also obtain a similar result (see Figure 3). By the comparison, it can be found that the greatest possible third family of compact stars may be the hybrid stars with the CFL quark core. Also, using the NL3 and GL85 EOS, we obtain the similar results. In the wide range of the CFL energy gap the third family of compact stars is found. The existence of the third family of compact stars may be equivalent to the existence of the phase transition from hadronic phase to the CFL quark phase.

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